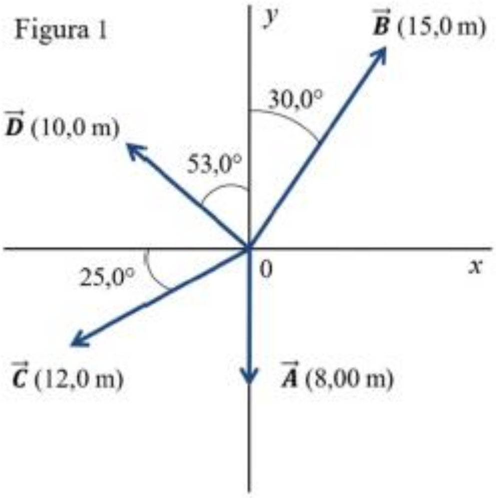
Práctica Unidad 1



1) Con los vectores $\vec{A}$ y $\vec{B}$ de la figura 1, use un dibujo a escala para obtener la magnitud y la dirección de:

a) La suma vectorial de $\vec{A} + \vec{B}$ y la diferencia $\vec{A} - \vec{B}$.

b) Con base en sus respuestas, determine la magnitud y la dirección de: $-\vec{A} - \vec{B}$ y $\vec{B} - \vec{A}$

Vector A:

$$A = (0m; -8m)$$

Vector B:

$$x_b = \text{sen}(30°)\ 15m = \cos(60°)\ 15m = 7{,}5m$$

$$y_b = \cos(30°)\ 15m = sen(60°)\ 15m = 12{,}99038106m$$

$$B = (7{,}5m; 13{,}0m)$$

a)i)

$$A + B = (0m; -8m) + (7{,}5m; 12{,}99038106m) = (7{,}5m; 4{,}99038106m)$$

$$|A + B| = \sqrt{(7{,}5m)^2 + (4{,}99038106m)^2} = 9{,}008546114m \cong 9{,}01m$$

$$\alpha^{a+b} = arc\ tg\left(\frac{4{,}99038106m}{7{,}5m}\right) = 33{,}63916434° = 33{,}6°$$

a)ii)

$$A - B = (0m; -8m) - (7{,}5m; 12{,}99038106m) = (-7{,}5m; -20{,}99038106m)$$

$$|A - B| = \sqrt{(-7{,}5m)^2 + (-20{,}99038106m)^2} = 22{,}29004479m = 22{,}3m$$

$$\alpha^{a-b} = arc\ tg\left(\frac{-20{,}99038106m}{-7{,}5m}\right) = 70{,}33785997° + 180° = 250{,}33785997° = 250°$$

b) i)

$$-A - B = -(0m; -8m) - (7{,}5m; 12{,}99038106m) = (-7{,}5m; -4{,}99038106m)$$

$$|-A - B| = \sqrt{(-7{,}5m)^2 + (-4{,}99038106m)^2} = 9{,}008546114m = 9{,}01m$$

$$\alpha^{-a-b} = arc\ tg\left(\frac{-4{,}99038106m}{-7{,}5m}\right) = 33{,}63916434° + 180° = 213{,}63916434° = 214°$$

b) ii)

$$B - A = (7{,}5m; 12{,}99038106m) - (0m; -8m) = (7{,}5m; 20{,}99038106m)$$

$$|B - A| = \sqrt{(7{,}5m)^2 + (20{,}99038106m)^2} = 22{,}29004479m = 22{,}3m$$

$$\alpha^{a-b} = arc\ tg\left(\frac{20{,}99038106m}{7{,}5m}\right) = 70{,}33785997° = 70°$$

2) Calcule las componentes x e y de los vectores $\vec{\boldsymbol{A}}, \vec{\boldsymbol{B}}, \vec{\boldsymbol{C}}$ y $\vec{\boldsymbol{D}}$ de la figura 1.

Vector A:

$$A = (0m; -8m)$$

Vector B:

$$x_b = \text{sen}(30°)\ 15m = \cos(60°)\ 15m = 7{,}5m$$

$$y_b = \cos(30°)\ 15m = sen(60°)\ 15m = 12{,}99038106m$$

$$B = (7{,}5m; 13{,}0m)$$

Vector C:

$$y_c = -\text{sen}(25°)\ 12m = -5{,}071419141m = -5{,}07m$$

$$x_c = -\cos(25°)\ 12m = -10{,}87569344m = -10{,}9m$$

$$C = (-10{,}9m; -5{,}07m)$$

Vector D:

$$x_d = -\operatorname{sen}(53°)\ 10m = -7{,}9863551m = -7{,}99m$$

$$y_d = \cos(53°)\ 10m = 6{,}018150232m = 6{,}02m$$

$$D = (-7{,}99m; 6{,}02m)$$

3) Para los vectores $\vec{A}$, $\vec{B}$ y $\vec{C}$ de la figura 1, obtenga los productos escalares.
a) $\vec{A} \cdot \vec{B}$
b) $\vec{B} \cdot \vec{C}$
c) $\vec{A} \cdot \vec{C}$

$$A = (0m; -8m) \qquad |A| = \sqrt{(0m)^2 + (-8m)^2} = 8m$$

$$B = (7{,}5m; 13{,}0m) \qquad |B| = \sqrt{(7{,}5m)^2 + (13m)^2} = 15m$$

$$C = (-10{,}9m; -5{,}07m) \qquad |C| = \sqrt{(-10{,}9m)^2 + (-5{,}07m)^2} = 12m$$

$$D = (-7{,}99m; 6{,}02m) \qquad |D| = \sqrt{(-7{,}99m)^2 + (6{,}02m)^2} = 10m$$

a)

$$A \,.\, B = |A|\ |B| \cos\alpha = 8m\ 15m \cos(150°) = -103{,}9230485m^2 = -104m^2$$

b)

$$B \,.\, C = |B|\ |C| \cos\alpha = 15m\ 12m \cos(145°) = -147{,}447368m^2 = -147m^2$$

c)

$$A \,.\, C = |A|\ |C| \cos\alpha = 8m\ 12m \cos(65°) = 40{,}571355313m^2 = 40{,}6m^2$$

4) Para los vectores $\vec{A}$ y $\vec{D}$ de la figura 1
a) Obtenga la magnitud y la dirección del producto vectorial $\vec{A}$ x $\vec{D}$.
b) Calcule la magnitud y la dirección de $\vec{D}$ x $\vec{A}$.

$$A\ x\ D = |A|\ |D| \operatorname{sen}\alpha = -8m\ 10m\ sen\ (127°) = -63{,}8908408m^2\ k = -63{,}9m^2\ k$$

$$D\ x\ A = |A|\ |D| \operatorname{sen}\alpha = 8m\ 10m\ sen\ (127°) = 63{,}8908408m^2\ k = 63{,}9m^2\ k$$

5) El vector $\vec{A}$ mide 2,80 cm y está 60,0º sobre el eje x en el primer cuadrante. El vector $\vec{B}$ mide 1,90 cm y está 60,0º bajo el eje x en el cuarto cuadrante de figura 2. Utilice las componentes para obtener la magnitud y la dirección de:

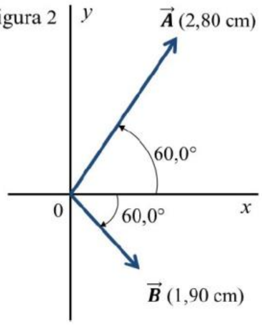
a) $\vec{A} + \vec{B}$

b) $\vec{A} - \vec{B}$

c) $\vec{B} - \vec{A}$.

En cada caso, dibuje la suma o resta de vectores, y demuestre que sus respuestas numéricas concuerdan cualitativamente con el dibujo.

Figura 2



a)

$$x_a = 2{,}80m\cos(60°) = 1{,}4m$$

$$y_a = 2{,}8m\,\text{sen}(60°) = 2{,}424871131m$$

$$A = (1{,}4m; 2{,}424871131m)$$

$$x_b = 1{,}9m\cos(60°) = 0{,}95m$$

$$y_b = -1{,}9m\,\text{sen}(60°) = -1{,}645448267m$$

$$B = (0{,}95m; -1{,}645448267m)$$

$$A + B = (1{,}4m; 2{,}424871131m) + (0{,}95m; -1{,}645448267m)$$
$$= (2{,}35m; 0{,}7794228634m)$$

$$|A + B| = \sqrt{(2{,}35m)^2 + (0{,}7794228634m)^2} = 2{,}475883681m = 2{,}48m$$

$$\alpha = arctg\left(\frac{0{,}7794228634m}{2{,}35m}\right) = 18{,}34909834° = 18{,}3°$$

b)

$$A - B = (1{,}4m; 2{,}424871131m) - (0{,}95m; -1{,}645448267m) = (0{,}55m; 4{,}070319398m)$$

$$|A - B| = \sqrt{(0{,}45m)^2 + (4{,}070319398m)^2} = 4{,}095119046m = 4{,}10m$$

$$\alpha = arctg\left(\frac{4{,}095119046m}{0{,}45m}\right) = 83{,}69120322° = 83{,}7°$$

c)

$$B - A = (0{,}95m; -1{,}645448267m) - (1{,}4m; 2{,}424871131m)$$
$$= (-0{,}55m; -4{,}070319398m)$$

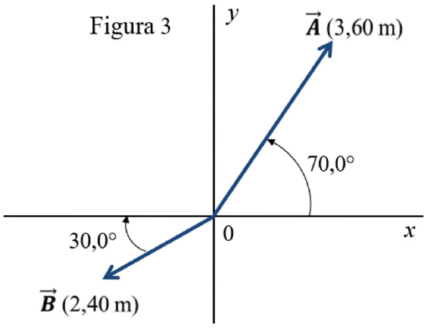
$$|A - B| = \sqrt{(-0{,}45m)^2 + (-4{,}070319398m)^2} = 4{,}095119046m = 4{,}10m$$

$$\alpha = arctg\left(\frac{-4{,}095119046m}{-0{,}45m}\right) = 83{,}69120322° + 180° = 263{,}69120322° = 264°$$

6) a) Escriba cada uno de los vectores de la figura 3 en términos de los vectores unitarios $\hat{\imath}$ y $\hat{\jmath}$.

b) Utilice vectores unitarios para expresar el vector $\vec{C}$, donde $\vec{C} = 3{,}00\ \vec{A} - 4{,}00\ \vec{B}$.

c) Determine la magnitud y la dirección de $\vec{C}$.



a)

Vector A:

$$y_a = \text{sen}(70°)\ 3{,}6m = 3{,}382893435m = 3{,}38m$$

$$x_a = \cos(70°)\ 3{,}6m = 1{,}231272516m = 1{,}23m$$

$$A = (1{,}231272516m; 3{,}382893435m)$$

$$A = (1{,}23m; 3{,}38m)$$

$$A = 1{,}23m\ i + 3{,}38m\ j$$

Vector B:

$$y_b = -\text{sen}(30°)\ 2{,}4m = -1{,}2m$$

$$x_b = -\cos(30°)\ 2{,}4m = -2{,}078460969m = -2{,}08m$$

$$B = (-2{,}078460969m; -1{,}2m)$$

$$B = (-2{,}08m ; -1{,}2m)$$

$$B = -2{,}08m\ i - 1{,}2m\ j$$

b)

$$C = 3{,}00\ A + 4{,}00\ B$$

$$C = 3{,}00\ (1{,}231272516m\ i + 3{,}382893435m\ j) - 4{,}00\ (-2{,}078460969m\ i - 1{,}2m\ j)$$

$$C = 3{,}693817548m\ i + 10{,}14868031m\ j + 8{,}3213843876m\ i + 4{,}8m\ j$$

$$C = 12{,}00766142m\ i + 14{,}94868031m\ j = 12{,}0m\ i + 14{,}9m\ j$$

c)

$$|C| = \sqrt{(12{,}00766142m)^2 + (14{,}9m)^2} = 19{,}17412256m = 19{,}2m$$

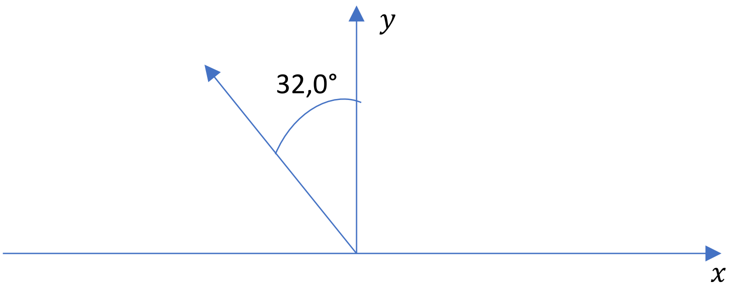
$$\alpha = arctg\left(\frac{14{,}94868031m}{12{,}00766142m}\right) = 51{,}22651607° = 51{,}2°$$

7) El vector $\vec{A}$, tiene una componente y $A_y = +13{,}0\ m$. $\vec{A}$ tiene un ángulo de 32,0° en sentido antihorario a partir del eje $+y$.

a) ¿Cuál es la componente x de $\vec{A}$?

b) ¿Cuál es la magnitud de $\vec{A}$?

a)



$$\frac{A_y}{h} = \frac{13m}{h} = cos\ (32°) \Rightarrow h = 15{,}32931924m$$

$$A_x = 15{,}32931924m\ sen\ (32°) = -8{,}123301575m = -8{,}12m$$

b)

$$|A| = \sqrt{(-8{,}123301575m)^2 + (13m)^2} = 15{,}32931924m = 15{,}3m$$

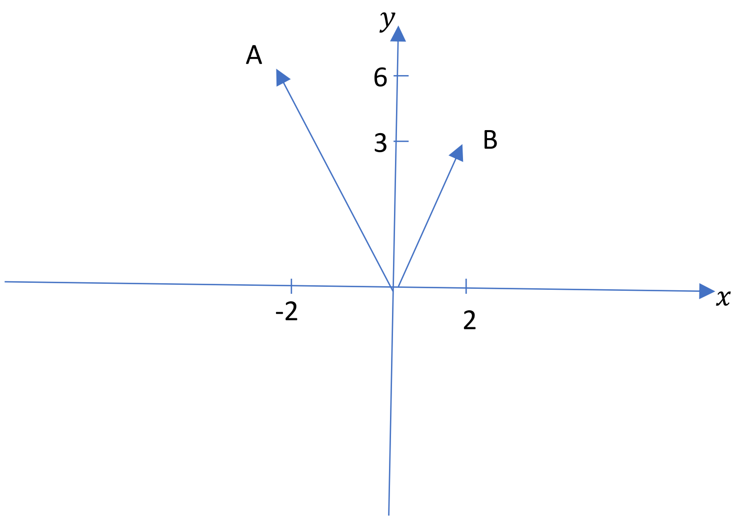
8) Calcule el ángulo entre estos pares de vectores:

a) $\vec{A} = -2{,}00\hat{\imath} + 6{,}00\,\hat{\jmath}$ y $\vec{B} = 2{,}00\hat{\imath} + 3{,}00\,\hat{\jmath}$

b) $\vec{A} = 3{,}00\hat{\imath} + 5{,}00\,\hat{\jmath}$ y $\vec{B} = 10{,}00\hat{\imath} + 6{,}00\,\hat{\jmath}$

c) $\vec{A} = -4{,}00\hat{\imath} + 2{,}00\,\hat{\jmath}$ y $\vec{B} = 7{,}00\hat{\imath} + 14{,}00\,\hat{\jmath}$

a)

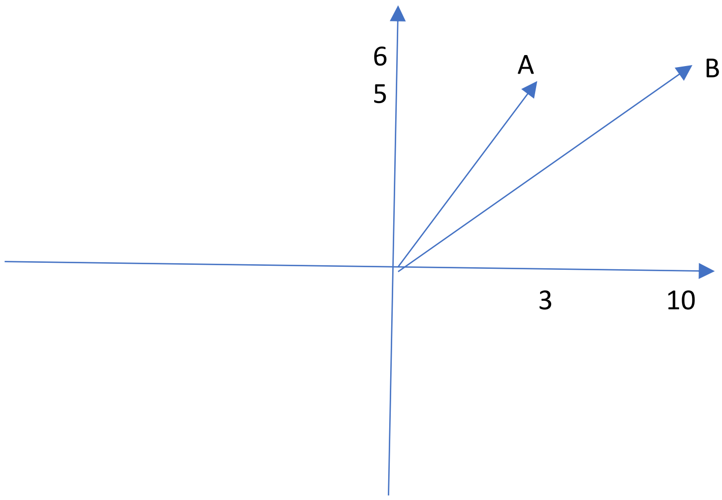


$$\alpha_b = arc\ tg\left(\frac{2}{3}\right) = 33{,}69006753°$$

$$\alpha_a = arc\ tg\left(\frac{2}{6}\right) = 18{,}43494882°$$

$$\alpha = 33{,}69006753° + 18{,}43494882° = 52{,}12501635° = 52{,}1$$

b)

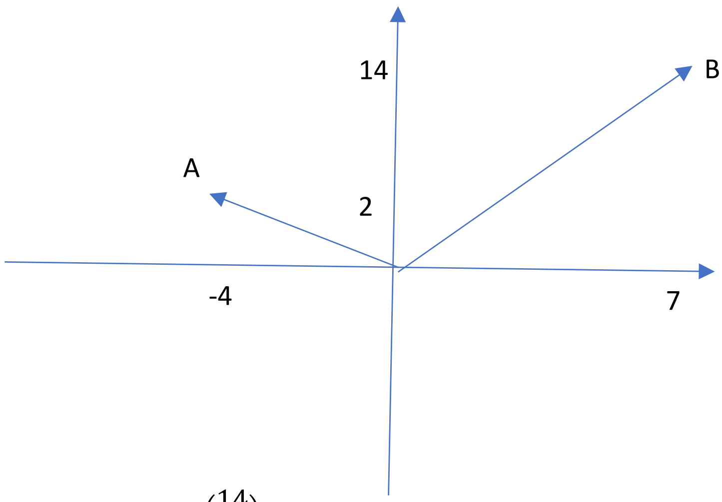


$$\alpha_b = arc\ tg\left(\frac{6}{10}\right) = 30{,}96375653°$$

$$\alpha_a = arc\ tg\left(\frac{5}{3}\right) = 59{,}03624347°$$

$$\alpha = 59{,}03624347° - 30{,}96375653° = 28{,}07248694° = 28{,}1°$$

c)



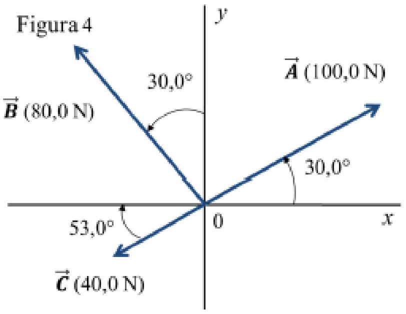
$$\alpha_b = arc\ tg\left(\frac{14}{7}\right) = 63{,}43494882°$$

$$\alpha_a = arc\ tg\left(\frac{4}{2}\right) + 90° = 153{,}43494882°$$

$$\alpha = 153{,}43494882° - 63{,}43494882° = 90°$$

9) Tres cuerdas horizontales tiran de una piedra grande enterrada en el suelo, produciendo los vectores de fuerza $\vec{A}$, $\vec{B}$ y $\vec{C}$ que se ilustra en la figura 4.

Obtenga la magnitud y la dirección de una cuarta fuerza aplicada a la piedra que haga que la suma vectorial de las cuatro fuerzas sea cero.



$$x_a = \cos(30°)\ 100N = 86{,}60254038N$$

$$y_a = \text{sen}(30°)\ 100N = 50N$$

$$A = (86{,}60254038N; 50N)$$

$$x_b = -\text{sen}(30°)\ 80N = -40N$$

$$y_b = \cos(30°)\ 80N = 69{,}2820323N$$

$$B = (-40N; 69{,}2820323N)$$

$$x_c = -\cos(53°)\ 40N = -24{,}07260093N$$

$$y_c = -\text{sen}(53°)\ 40N = -31{,}9454204N$$

$$C = (-24{,}07260093N; -31{,}9454204N)$$

$$(86{,}60254038N; 50N) + (-40N; 69{,}2820323N) + (-24{,}07260093N; -31{,}9454204N) + D = (0; 0)$$

$$(22{,}52993945N; 87{,}3366119N) + D = (0; 0)$$

$$D = (-22{,}52993945N; -87{,}3366119N)$$

$$|D| = \sqrt{(-22{,}52993945N)^2 + (-87{,}3366119N)^2} = 90{,}19579785N = 90{,}2N$$

$$\alpha = arc\ tg\left(\frac{-87{,}3366119N}{-22{,}52993945N}\right) = 75{,}53495938° + 180° = 255{,}53495938° = 256°$$